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ABSTRACT

The study investigated the effect of instruction on basic number fact mastery of 123 learning disabled (LD) students, 8 to 13 years old. Mastery of basic addition, subtraction, and multiplication facts was seen as an important component of overall arithmetic competence and represents a particular area of performance deficit among LD students. The intervention study compared two treatment approaches containing parallel program features, but with differences in the sequential clusters of facts taught. Instruction on each cluster of facts involved four phases: (1) presentation of activities (during group lessons the number facts in the cluster were explored using concrete materials and/or graphic representations); (2) developmental activities (active practice of fact clusters emphasized accuracy without regard to speed with teachers supervising pairs or small groups of students playing one or more games, performing oral or blackboard reviews, or individuals working on activity sheets); (3) mastery activities (practice of fact clusters emphasizing rapid, automatic responses); and (4) criterion testing (students had to meet a preestablished criterion before moving from one cluster of facts to another). Basic Fact Sequence 1 followed traditional grouping and sequencing, while Basic Fact Sequence 2 grouped and ordered related facts according to "thinking" strategies. Results of pre-, post-, and retention tests, each 3-minute written power tests of basic fact proficiency were analyzed. Significant gains were made on post-tests; these gains were maintained during a 6-week uninstructed period. No effect was found for Basic Fact Sequences, leading to the conclusion that either sequence is effective in promoting mastery of basic facts under the instructional conditions utilized. (Author/SW)

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MASTERY OF BASIC NUMBER FACTS BY LEARNING DISABLED STUDENTS:
AN INTERVENTION STUDY

Jeannette E. Fleischner, Katherine Garnett, and David Preddy

Technical Report #17

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Abstract

This study investigated the effect of instruction on basic number fact mastery of a group of 125 learning disabled (LD) students, 8 to 13 years. Mastery of basic addition, subtraction, and multiplication facts is an important component of overall arithmetic competence and represents a particular area of performance deficit among LD students. This intervention study compared two treatment approaches, containing parallel program features (game-format drills, mastery learning and self-charting of progress). The approaches differed in grouping and sequencing of facts presented. Basic Fact Sequence-1 (BFS-1) followed traditional grouping and sequencing, while Basic Fact Sequence-2 (BFS-2) grouped and ordered related facts according to "thinking" strategies. Results of pre-, post-, and retention tests, each three-minute written power tests of basic fact proficiency, were analyzed through repeated measures ANOVAs. Significant gains were made on post-tests; these gains were maintained during a six-week uninstructed periods. No effect was found for Basic Fact Sequences, leading to the conclusion that either sequence is effective in promoting mastery of basic facts under the instructional conditions described.

Mastery of Basic Number Facts by Learning Disabled Students:

An Intervention Study

Basic texts in learning disabilities mourn the paucity of literature on the nature and treatment of arithmetic learning disabilities (Bryan and Bryan, 1978; Hammill and Bartel, 1978; Lerner, 1981). Arithmetic is defined as a "...branch of mathematics that deals with real numbers and their computation..." (Chalfant and Scheffelin, 1969, p. 119). The development of computational facility is a major goal of the mathematics curriculum during the elementary school years. In fact, it has been argued that computational facility in addition, subtraction, multiplication and division is a necessary, although not sufficient, condition for mathematical competence (National Council of Teachers of Mathematics Yearbook, 1978).

One of the first indicators of progress toward mathematical competence is the mastery of basic number facts. These are addition problems up to and including $9 + 9$; subtraction problems up to and including $18 - 9$; multiplication problems up to and including 9×9 ; and division problems up to and including $81 \div 9$. Included are 100 separate facts for each of the four arithmetic operations. Proficiency in computation of these basic facts has been considered to be fundamental to adequate arithmetic achievement (Ashlock and Washbourne, 1978; Suydam and Dessart, 1976). Children with learning disabilities have long been viewed by their teachers as having serious difficulty in mastering basic facts. Empirical investigation has confirmed that learning disabled youngsters are significantly less proficient than their non-disabled peers on measures of basic fact computation (Fleischner, Garnett and Shepherd, 1960).

Computing basic facts depends on the use of either a reproductive or reconstructive strategy (Groen and Parkman, 1972). For example, when asked "How much is $8 \div 6$?", many may respond rapidly by reproducing the answer, retrieving it directly from long-term memory; others may reconstruct the answer, using one or another mediating strategy based on counting. Reconstructive strategies may be either inefficient--for example, counting $8+1+1+1+1+1+1$ to find the answer, or may be sophisticated and efficient--for example, using known facts as "anchor points" to limit the need for counting. Children's strategies for computing basic facts develop with age and practice. By the end of the elementary school years, normally achieving children seem to use a combination of a reproductive process and the more sophisticated and efficient reconstructive strategies. In a sense, consistent use of this combination of reproductive and efficient reconstructive strategies can be considered the hallmark of "knowing" basic number facts-- being proficient, or having attained mastery.

Instruction in basic fact computation generally follows a fixed pattern: addition facts whose sums range from zero to five are taught, followed by subtraction facts whose remainder is from five to zero. The next grouping includes facts 6 - 10, and finally the facts from 11 - 18 are taught. A similar pattern, from smaller to larger combinations, is employed in teaching multiplication facts. The sequencing of facts based on the magnitude of sum, difference or product, constitutes the traditional sequence for teaching number facts (Fleischner and Garnett, 1979). The rationale for arranging facts in this sequence derives from seminal investigations into the relative difficulty of

of the different number facts (Washburne and Vogel, 1928; Clapp, 1934; Knight and Behrens, 1928).

Instruction based on facts grouped in a different sequence has been proposed over the years (Brownell, 1935; Rathmell, 1978; Swenson, 1949; Thornton, 1978). This non-traditional grouping of facts is based on the relationships among the numbers themselves, and on observations of specific reconstructive strategies which are commonly used in computing particular facts (Brownell and Carper, 1943; Jerman, 1970; Woods, Resnick and Groen, 1975). For example, children may count on, make use of doubles, employ the commutative property, or use certain combinations as "anchors" for computing less well-known facts. Additionally, it would appear that certain combinations have heuristics associated with them which are presumed to facilitate their retention and swift retrieval (e.g. $n + 0$, $n + n$, $n \times 5$). This facilitation effect is observed in the comparatively lower response latencies for combinations of these types (Jerman, 1970). Recent investigations have suggested that, both for non-disabled children (Carnine and Stein, 1981; Rathmell, 1978; Thornton, 1978) and for learning disabled children (Myers and Thornton, 1977), mastery may be facilitated when instruction makes explicit the relationships among basic facts.

Method

Design

The purpose of this study was to investigate the extent to which learning disabled children's basic fact computational facility could be improved through short-term, systematic instruction, to determine

whether gains would be maintained after instruction was terminated, and to compare whether differences in the sequencing and grouping of facts presented would affect the degree of proficiency attained. Subjects' (Ss) written computational performance was assessed on pre-test, post-test, and six-week retention test measures. An eight-week long instructional program consisting of three 20-minute sessions each week, was carried out by the teachers of intact classes which were randomly assigned to Basic Facts Sequence 1 (BFS-1) or Basic Facts Sequence 2 (BFS-2). Total instructional time was eight hours. Subjects received instruction either in addition/subtraction or in multiplication, but not in both topics.

In order to assess the effectiveness of instruction overall, pre-to-post test, and pre-to-retention test gain scores were compared. To assess whether there was differential benefit from one or the other instructional sequence, and to explore the various possible interactions, analysis of variance techniques were used.

Subjects

Subjects (Ss) for this study were 126 learning disabled (LD) children, ranging in age from 8 to 13 years. They were enrolled in 23 self-contained 3rd through 6th grade classes within three private day schools for LD students in the New York metropolitan area. All Ss were classified as learning disabled under the regulations of New York or New Jersey. Information on IQ scores was available for 105 subjects (84%). Mean IQ was 96, with a standard deviation of 14.3. These LD students showed significant discrepancies between expected performance levels in reading and arithmetic and actual achievement

levels. Performance deficits of two or more years were commonly reported for these students.

Two criteria were used in selecting Ss for this study: 1) performance on basic fact proficiency pretests had to fall at least one and a half standard deviations (SD) below mean performance levels of non-disabled children in comparable grades (Fleischner, et al., 1980); 2) Ss had to demonstrate understanding of the concepts of addition, subtraction, and multiplication by manipulating blocks correctly to "prove" an equation such as $7 + 3 = 10$, and had to write numbers sentences to dictation.

Basic Fact Proficiency Tests

These tests were designed to measure the speed and accuracy of written responses to basic fact arithmetic problems. Three separate 98-problem tests were used, one for each operation (addition, subtraction, and multiplication). Problems were printed in bold primary type, in vertical format, and were randomly sequenced on two pages. All subjects were given the addition and subtraction tests; only the 5th and 6th graders received the multiplication measure. Three minutes were allowed for completion of each test.

All tests administered were completed during one session, and order of presentation was counterbalanced. Tests were scored with both the number attempted and the number correct noted. The same problems were used for pre-tests, post-tests and retention-tests, although they were sequenced differently for each administration.

The Instructional Programs

The two instructional programs required the use of a number of materials which were produced by the investigators¹. Two different groupings of facts were employed: BFS-1 relied on the traditional sequence of facts based on magnitude of sum, difference or product. BFS-2 referred to the grouping of facts according to the thinking strategies applied in reconstructing those facts. For instance, facts such as $2 + 2$, $8 + 8$, $4 + 4$ were grouped together because they were "doubles". Similarly, $2 + 3$, $8 + 9$ and $4 + 5$, were grouped together because they were "doubles + 1". Figure 1 presents the instructional sequences of facts as they were presented for addition, subtraction, and multiplication.

Insert Figure 1 about here

The instructional programs employed parallel methodologies and materials, but differed in the sequential clusters of facts taught. Instruction on each cluster of facts involved four phases:

- 1) Presentation Activities: During group lessons the number facts in the cluster were explored using concrete materials and/or graphic representations (e.g., blocks, abacus, counters, pictures, etc.) The commutative principle was emphasized throughout the teaching of addition and multiplication.

- 2) Developmental Activities: Active practice of fact clusters emphasized accuracy without regard to speed. Teachers supervised

as pairs or small groups of subjects played one or more games, performed oral or blackboard reviews, or as individuals worked on activity sheets.

3) Mastery Activities: Practice of fact clusters now emphasized rapid, automatic responses. As during developmental activities, subjects worked alone, in pairs, or in small groups, playing fact games, completing activity sheets, and/or engaging in flash card drills, but all activities focused on increasing speed without sacrificing accuracy.

4) Criterion Testing: Students had to meet a pre-established criterion before moving from one cluster of facts to another. This criterion was a perfect score on a ten-problem test sheet completed within 30 seconds on each of two successive days. After criterion was reached, subjects repeated the instructional phases using each subsequent cluster with the addition of cumulative review as a consistent feature of the programs.

Several principles guided the design of the instructional programs and accompanying materials. These principles derived from conceptions about teaching and learning, and the needs of LD students. Key features of the instructional programs which related to these principles included: a) use of a variety of presentation models and practice formats; b) active student participation in manipulating objects, constructing problems, and monitoring the accuracy of responses during games; c) individual student responsibility for charting progress; d) mastery of each basic fact cluster before beginning a new unit; and, e) cumulative review to insure that facts

learned were retained.

Materials

Teachers were provided with an instructional package containing a teaching guide and student activity sheets, as well as games and activities for use by partners and small groups during the Developmental and Mastery phases. The interactive games and activities included board games, roll games, and card games. Typical of these were:

1) General board games: These were several start-to-finish paths that could be used with any cluster of facts. To play, the child drew a card and rolled a die or spun a spinner; on the card was a basic fact problem (+, -, or X); if the child answered correctly, she/he moved the number of spaces on the die or spinner. The first child to complete the pathway was the winner.

2) Roll games: On an 8 1/2" x 11" sheet, 12 basic fact problems were listed (6 problems and their related commutatives). Down each side of the page were 12 small circles, one next to each problem. Students, in turn, threw a die, then placed a marker over the circle adjoining the problem whose answer was displayed on the die. The winner was the first child to cover all 12 circles.

3) Card games: These games were played with "cluster" decks which had each problem in the cluster represented on two cards and each solution on two cards. Two types of card games were played: War and Concentration. War was played by splitting the deck evenly between two or more players; each player turned over the top card simultaneously. The player with the answer of greatest magnitude won all cards. The player with the most cards at the end of the round.

was the winner.

To play Concentration, each basic fact problem and its solution was arrayed randomly, blank side up. In turn, each player turned over two cards. If those turned up were a problem and its match or its solution, the player retained the cards; if the two cards did not match, they were replaced in the array, face down. When all cards had been matched, the player with the most cards was declared the winner.

Materials of different types of games were developed for each instructional cluster. Cards for a given cluster of facts could be used with many basic boards, as well as being used for War, Concentration, or Go Fish. This variety during the developmental phase was provided in order to maintain student interest in the activities.

Results

This study had two purposes. The first was to determine whether the speed of accurate recall of basic facts by LD⁸ students could be improved through systematic instruction. The second was to investigate whether it was differentially beneficial to sequence basic facts during instruction according to commonly used reconstructive strategies, rather than in the traditional manner.

Effect of Instruction

In order to determine whether instruction was effective in improving basic fact performance scores, a repeated measures ANOVA was computed on gains scores from pre- to post- to retention tests. (See Table 1). Results indicate that there was an effect of instruc-

tion when pre-test to post-test scores were considered ($F = 441.3$

Insert Table 1 about here

$df = 1$, $p < .001$) and when pre- to post- to retention test scores were considered ($F = 91.8$, $df = 2$, $p < .001$). On the average, and regardless of grade level, subjects made significant improvement from pre-test to post-test. Scores on the retention test demonstrate that this gain was maintained through a six-week period when no special instruction was provided.

Effect of Basic Fact Sequence

Performance of Ss in the BFS-1 program was compared to that of Ss in the BFS-2 program through a repeated measures ANOVA on gain scores from pre-test to post-test to retention test. Results, reported

Insert Table 2 about here

in Table 2, again reveal a significant effect of instruction ($F = 370.4$, $df = 1$, $p < .001$), but fail to reveal any difference between performance scores of students whose program emphasized traditional arrays of facts (BFS-1) or arrays of facts organized according to strategies commonly used to reconstruct them (BFS-2). Thus, while it can be said that instruction was beneficial in improving basic fact recall performance, it cannot be said that the way in which facts were sequenced in instruction and practice had any effect on the rate of gain made during an eight-week long instructional program, or any effect on the maintenance

of those gains.

Discussion

Learning disabled students have been found to be less proficient at all computation tasks than are their non-disabled peers. One important factor in this general arithmetic performance deficit would seem to be inadequate mastery of basic number facts. A major goal of arithmetic instruction for LD students, after they have mastered the concepts implicit in the fundamental processes of addition, subtraction, multiplication and division, should be to increase the speed and accuracy of recall of basic facts, so that time and effort in more complex calculation is not expended on laborious and repeated recomputing.

The results of this study indicate that this is a reasonable goal. Significant gains in performance scores on basic fact tests were obtained through an instructional program conducted for three twenty-minute periods for eight weeks. Furthermore, these gains were maintained over a six-week period when no special emphasis was placed on basic fact mastery. This finding is consistent with results reported in studies investigating rate of progress and retention of information by LD students taught reading skills through highly systematic instruction (Bryant, Fayne and Gettinger, 1980(a); 1980(b)). In the present study, as in others, LD students were found to have good long-term retention of information that had been mastered. This finding certainly supports the efficacy of continued instruction aimed at mastery of basic facts. It was interesting to note how many LD students seemed unaware that they should be able to respond rapidly to basic fact problems. They accurately

used lengthy counting-on procedures to solve such problems, and in the course of the study, often were heard to comment that they hadn't realized that they could learn to associate a number combination directly with a correct answer.

A second important purpose of this study was to investigate whether LD students' rate of gain on basic fact tests was influenced by the way in which facts were sequenced for instructional practice. Recently, it has been asserted that there are significant benefits to making explicit the relationship among basic facts in order to emphasize the heuristic properties of certain combinations (Carnine and Stein, 1981; Meyers and Thornton, 1977; Rathwell, 1978; Thornton, 1978).

In fact, Carnine and Stein (1981) and Thornton (1978) report that instruction in which related facts were grouped, and those relationships were emphasized, resulted in increased rate of mastery and durability of retention of basic facts. However, small numbers of subjects (Carnine and Stein, 1981) and failure to provide comparable programs to traditionally and experimentally taught groups (Thornton, 1978) make these findings questionable.

In the present study, rate of gain was comparable under both instructional conditions. These LD students performed significantly better on post-test and retention tests of basic facts regardless of whether facts were presented in the traditional sequence (according to magnitude of sum or difference in addition or subtraction, or to the order of ascending tables in multiplication) or in a sequence

emphasizing interfact relations and the heuristics associated with certain combinations.

References

- Ashlock, R. B., & Washbon, C. A. Games: Practice activities for the basic facts. National Council of Teachers of Mathematics Yearbook. Reston, VA.: National Council of Teachers, 1978.
- Brownell, W. A. Psychological considerations in the learning and teaching of arithmetic. The teaching of arithmetic. Tenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Columbia University, 1935.
- Brownell, W. A., & Carper, D. V. Learning the multiplication combinations. Duke University Studies in Education, No. 7. Durham, NC: Duke University Press, 1943.
- Bryan, T. H., & Bryan, J. H. Understanding learning disabilities (2nd ed.). Sherman Oaks, CA: Alfred Publishing Co., 1978.
- Bryant, N. D., Fayne, H. R., & Gettinger, M. LD-Efficient instruction in phonics applying sound learning principles to remedial teaching. New York: Teachers College, Columbia University, 1980. ERIC # ED 210 832.
- Carnine, D. W., & Stein, M. Organizational strategies and practice procedures for teaching basic facts. Journal for Research in Mathematics Education, 1981, 12, 65-69.
- Chalfant, J. C., & Sheffelin, M. A. Central processing dysfunctions in children: A review of research. Bethesda, MD: U.S. Department of Health, Education, and Welfare, 1969.
- Clapp, F. L. The number combinations, their relative difficulty and the frequency of their appearance in textbooks. University of Wisconsin Bureau of Education Research, 1-2, 1934.

Fleischner, J. E., & Garnett, K., Arithmetic learning disabilities :

A literature review. Teachers College, Columbia University,

Research Institute for the Study of Learning Disabilities,

Research Review Series 4, 1977-80. ERIC #ED 210 843.

Fleischner, J. E., Garnett, K., & Shepherd, M. J. Proficiency in arithmetic basic fact computation of learning disabled and non-disabled children. New York: Teachers College, Columbia University, Research Institute for the Study of Learning Disabilities, Technical Report 9, 1980. ERIC #ED 210 838.

Groen, G., & Parkman, J. A chronometric analysis of simple addition.

Psychological Review, 1972, 79, 329-343.

Hammill, D. D., & Bartel, N. R. Teaching children with learning and behavior problems (2nd ed.). Boston: Allyn and Bacon, Inc., 1978.

Jerman, M. Some strategies for solving simple multiplication combinations. Journal for Research in Mathematics Education, 1970, 1, 95-128.

Knight, F. B., & Behrens, M. S. The learning of the 100 addition combinations and the 100 subtraction combinations. New York: Longmans, Green, 1928.

Lerner, J. W. Children with learning disabilities (2nd ed.) Boston: Houghton Mifflin, 1981.

Myers, A. C., & Thornton, C. A. The learning disabled child: Learning the basic facts. Arithmetic Teacher, 1977, 24, 46-50.

National Council of Teachers of Mathematics, Yearbook 1978. Reston, VA:

National Council of Teachers of Mathematics, 1978.

Rathmell, E. C. Using thinking strategies to learn the basic facts.

In M. Suydam (Ed.), 1978 Yearbook of the National Council of Teachers of Mathematics. Reston, VA: National Council of Teachers of Mathematics, 1978.

Spydam, M. N., & Dessart, D. J. Classroom ideas from research on computational skills. Reston, VA: National Council of Teachers of Mathematics, 1976.

Swenson, E. J. Organization and generalization as factors in learning, transfer, and retroactive inhibition. Learning theory in school situations. University of Minnesota Studies in Education, No. 2. Minneapolis, MN: University of Minnesota Press, 1949.

Thornton, C. A. Emphasizing thinking strategies in basic fact instruction. Journal for Research in Mathematics Education, 1978, 214-227.

Washbourne, C., & Vogel, M. Are any number combinations inherently difficult? Journal of Educational Research, 1928, 17, 235-255.

Woods, S. S., Rasnick, L. B., & Groen, G. J. An experimental test of five process models for subtraction. Journal of Educational Psychology, 1975, 67, 17-21.

Footnotes

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Table I

Means, Standard Deviations (by Grade) and Analysis of Variance
of Pre-test, Post-test, and Retention Test Performance Scores
on a Timed Test of Basic Facts

GRADE		PRE-TEST	POST-TEST	RETEST
3 (n=36)	mean	16.3	29.2	27.6
	S.D.	14.2	20.4	21.0
4 (n=25)	mean	21.6	34.8	38.6
	S.D.	15.6	16.2	18.9
5 (n=21)	mean	29.0	50.4	48.8
	S.D.	13.8	19.4	20.9
6 (n=43)	mean	39.3	57.9	57.7
	S.D.	21.6	26.8	28.5
TOTAL (n=125)	mean	27.4	43.7	43.7
	S.D.	19.6	25.1	26.3

Analysis of Variance

SOURCE	SS	df	MS	F	Sig
Pre-Post	489611.8	1	489611.8	441.3	.001
Pre-Retest	280554.2	1	280554.2	392.2	.001
Pre-Post-Retest	21144.6	2	10572.3	91.8	.001
Pre-Post/Grade	29990.3	3	9996.7	14.69	.001
Pre-Retest/Grade	15941.0	3	676.7	113.8	.001

Table II

Means, Standard Deviations and Analysis of Variance
on Pre-test, Post-test and Retention Test Performance Scores
by Basic Fact Sequence

<u>Basic Fact Sequence</u>		PRE-TEST	POST-TEST	RETEST
BFS-1 (Traditional) (n = 63)	mean	27.2	43.7	45.7
	S.D.	19.4	23.2	26.5
BFS-2 (Thinking Strategies) (n = 62)	mean	27.6	43.8	41.7
	S.D.	19.9	27.0	26.2

Analysis of Variance

SOURCE	SS	df	MS	F	Sig
Pre-Post	550030.4	1	550030.4	370.4	.000
Pre-Post/by method	125.6	1	125.6	0.08	.772
Pre-Post-Retest	22201.4	2	11100.7	95.5	.000
Pre-Post-Retest/by method	376.7	2	188.3	1.62	.208

Figure 1. Sequence of Basic Facts Presented

Addition		Subtraction		Multiplication	
BFS-1	BFS-2	BFS-1	BFS-2	BFS-1	BFS-2
1) facts 0 - 5	1) doubles (2+2, 3+3, etc.)	1) 0-0 to 5-5	1) doubles subtraction (2-1, 4-2, 6-3, etc.)	1) x 0	1) x 2
2) facts 6 - 10	2) doubles + 1 (addends differ by 1: 2+3, 3+4, etc.)	2) 6-0 to 8-8	2) doubles + 1 subtraction (3-1, 5-2, 17-9, etc.)	2) x 1	2) x 5
3) facts 11 - 14	3) doubles + 2 (addends differ by 2: 2+4, 3+5, etc.)	3) 9-0 to 10-9	3) doubles + 2 subtractions (4-1, 4-3, 16-7, etc.)	3) x 2	3) x 0, x 1
4) facts 15 - 18	Only sums 2 - 18 were included	4) 11-2 to 12-9	4) subtracting 10	4) x 3	4) 9
	4) adding 10 (2+10, 5+10, etc.)	5) 13-4 to 14-9	5) subtracting 9 (taught as $[N-10] \pm 1$)	5) x 4	5) perfect square (1x1, 3x3, 6x6, etc.)
	5) adding 9 (taught as $[N+10] - 1$)	6) 15-6 to 18-9	6) remaining subtraction facts (7-2, 7-5, 8-2, 8-6, 9-3, 9-6, 10-3, 10-7, 11-3, 11-8, 11-4, 11-7)	6) x 5	6) remaining multiplication facts (3x4, 3x6, 3x7, 3x8, 4x6, 4x7, 4x8, 6x7, 6x8, 7x8)
	6) remaining addition facts (facts which do not fit other categories: 2+5, 2+6, 2+7, 2+8, 3+6, 3+7, 3+8, 4+7, 4+8, 5+8)			7) x 6	
				8) x 7	
				9) x 8	
				10) x 9	